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Bianchi Type-III Cosmological Model with Negative Constant Deceleration Parameter in Brans Dicke Theory of Gravitation

K.S. Adhav · A.S. Nimkar · M.R. Ugale · M.V. Dawande

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Abstract Bianchi type-III space time is considered in the presence of perfect fluid source in the scalar-tensor theory of gravitation proposed by Brans and Dicke (Phys. Rev. 124:925, 1961). With the help of special law of variation for Hubble's parameter proposed by Bermann (Nuovo Cimento 74B:182, 1983) a cosmological model with negative constant deceleration parameter is obtained in the presence of perfect fluid with disordered radiation. Some physical and kinematical properties of the model are also discussed.

Keywords Radiating model · Negative constant deceleration parameter · Brans-Dicke theory · Bianchi type-III space time

1 Introduction

Einstein's general theory of relativity has been very successful in describing gravitational phenomena. It has also served as a basis for models of the universe. However, since Einstein first published his theory of gravitation there have been many criticisms of general relativity because of the lack of certain 'desirable' features in the theory. For example, Einstein himself pointed out that general relativity does not account satisfactorily for inertial properties of matter i.e. Mach's principle is not substantiated by general relativity. So, in recent years, several theories of gravitation have been proposed as alternatives for Einstein's theory. The most important among them are scalar-tensor theories of gravitation formulated by Jordan [3], Brans and Dicke [1], Nordtvedt [4], Ross [5] and Schmidt et al. [6]. Brans-Dicke [1] theory of gravitation is the simplest example of scalar-tensor theories in which a scalar field ϕ has been introduced in addition to the familiar general relativistic metric tensor. In this theory the scalar field has the dimension of inverse of the gravitational constant and its role is confined to its effects on gravitational equations.

K.S. Adhav (✉) · A.S. Nimkar · M.R. Ugale · M.V. Dawande
Department of Mathematics, S.G.B. Amravati University, Amravati, Maharashtra, India
e-mail: ati_ksadhav@yahoo.co.in

Brans-Dicke [1] field equations for combined scalar and tensor fields are

$$G_{ij} = -8\pi\phi^{-1}T_{ij} - \omega\phi^{-2}\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{'k}\right) - \phi^{-1}(\phi_{i,j} - g_{ij}\Box\phi) \tag{1}$$

and

$$\Box\phi = \phi^{'k}_{;k} = 8\pi\phi^{-1}(3+2\omega)^{-1}T, \tag{2}$$

where $G_{ij} = R_{ij} - \frac{1}{2}g_{ij}R$ is the Einstein tensor, T_{ij} is energy tensor of the matter, ω is the dimensionless coupling constant, comma and semicolon denote partial and co-variant differentiation respectively.

The equations of motion

$$T^{ij}_{;j} = 0 \tag{3}$$

are consequences of the field equations (1) and (2).

The study of Brans-Dicke cosmology in the presence of perfect fluid has attracted the attention of many workers. The work of Singh and Rai [7] gives a detailed survey of Brans-Dicke cosmological models discussed by several authors. In particular Singh and Rai [8] obtained solutions of Brans-Dicke field equations when the source of gravitational field is a perfect fluid with pressure equal to energy density and the metric is cylindrically symmetric of Marder type. Bermann [2] presented a law of variation of Hubble's parameter and obtained some cosmological models within the frame work of Saez-Ballester [9] scalar-tensor theory of gravitation.

Recently, Rahaman et al. [10] studied some cosmological models in Lyra geometry [11] using a special law of variation of Hubble's parameter that yields constant deceleration parameter models of the universe while Reddy and Venkateswara Rao [12], Reddy et al. [13–15] discussed Bianchi type-I cosmological models with negative constant deceleration parameter in scalar-tensor theories formulated by Brans-Dicke [1] and Saez-Ballester [9].

In this paper, we discuss Bianchi type-III radiating cosmological model with negative constant deceleration parameter in the scalar tensor theory of gravitation proposed by Brans-Dicke [1] in the presence of perfect fluid.

2 Field Equations and the Model

We consider the general Bianchi type-III space time with the line element

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)e^{-2ax}dy^2 - C^2(t)dz^2, \tag{4}$$

where 'a' is a constant.

We have the perfect fluid energy momentum tensor as

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \tag{5}$$

together with

$$g_{ij}u^i u_i = 1 \tag{6}$$

where u^i is the four velocity vector of the fluid and p and ρ are the proper pressure and energy density respectively. From (4) to (6) the components of T^i_j in comoving coordinates are

$$T^1_1 = T^2_2 = T^3_3 = -p, \qquad T^4_4 = \rho, \qquad T = \rho - 3p. \tag{7}$$

Now the Brans-Dicke field equations (1) and (2) for the metric (4) with the help of (5) to (7) can be written as

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_{44}}{\phi}+\frac{\phi_4}{\phi}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)=-8\pi\phi^{-1}p, \tag{8}$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}+\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_4}{\phi}\left(\frac{A_4}{A}+\frac{C_4}{C}\right)+\frac{\phi_{44}}{\phi}=-8\pi\phi^{-1}p, \tag{9}$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{a^2}{A^2}+\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_4}{\phi}\left(\frac{A_4}{A}+\frac{B_4}{B}\right)+\frac{\phi_{44}}{\phi}=-8\pi\phi^{-1}p, \tag{10}$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{a^2}{A^2}-\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_4}{\phi}\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=8\pi\phi^{-1}\rho, \tag{11}$$

$$\frac{A_4}{A}-\frac{B_4}{B}=0, \tag{12}$$

$$\phi^{i}_{;i}=\phi_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4=\frac{-8\pi\phi^{-1}}{(3+2\omega)}(\rho-3p). \tag{13}$$

Equation (3) which is a consequence of the field equations takes the form

$$\rho_4+(\rho+p)\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=0. \tag{14}$$

From (12), without loss of generality, we get

$$A=B. \tag{15}$$

Using (15), the above (8) to (14) reduce to

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_4}{\phi}\left(\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{\phi_{44}}{\phi}=-8\pi\phi^{-1}p, \tag{16}$$

$$2\frac{B_{44}}{B}+\left(\frac{B_4}{B}\right)^2-\frac{a^2}{B^2}+\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+2\frac{\phi_4}{\phi}\frac{B_4}{B}+\frac{\phi_{44}}{\phi}=-8\pi\phi^{-1}p, \tag{17}$$

$$\left(\frac{B_4}{B}\right)^2+2\frac{B_4C_4}{BC}-\frac{a^2}{B^2}-\frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2+\frac{\phi_4}{\phi}\left(2\frac{B_4}{B}+\frac{C_4}{C}\right)+\frac{\phi_{44}}{\phi}=8\pi\phi^{-1}\rho, \tag{18}$$

$$\phi_{44}+\left(2\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4=\frac{8\pi\phi^{-1}}{(3+2\omega)}(\rho-3p), \tag{19}$$

$$\rho_4+(\rho+p)\left(2\frac{B_4}{B}+\frac{C_4}{C}\right)=0 \tag{20}$$

where the suffix 4 following an unknown function denotes ordinary differentiation with respect to time t. Equations (16) to (19) are four independent equations in five unknowns B, C, ρ, p and ϕ. To get a determinate solution, one extra condition is needed. So we consider the equation of state

$$\rho = 3p \tag{21}$$

which represents matter distribution with disordered radiation.

Also the set of equations being highly non-linear, we assume a relation between metric coefficients given by

$$B = \mu C$$

where μ is constant.

With the help of (21), the set of equations (16–20) reduces to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4 C_4}{BC} + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 - \frac{B_4}{B}\frac{\phi_4}{\phi} = -8\pi\phi^{-1}p, \tag{22}$$

$$2\frac{B_{44}}{B} + \left(\frac{B_4}{B}\right)^2 - \frac{a^2}{B^2} + \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 - \frac{C_4}{C}\frac{\phi_4}{\phi} = -8\pi\phi^{-1}p, \tag{23}$$

$$\left(\frac{B_4}{B}\right)^2 + 2\frac{B_4 C_4}{BC} - \frac{a^2}{B^2} - \frac{\omega}{2}\left(\frac{\phi_4}{\phi}\right)^2 + 2\frac{\phi_4}{\phi}\frac{B_4}{B} + \frac{C_4}{C}\frac{\phi_4}{\phi} = 8\pi\phi^{-1}\rho, \tag{24}$$

$$\phi_{44} + \left(2\frac{B_4}{B} + \frac{C_4}{C}\right)\phi_4 = 0, \tag{25}$$

$$\rho_4 + \frac{4}{3}\rho\left(2\frac{B_4}{B} + \frac{C_4}{C}\right) = 0. \tag{26}$$

We solve the above set of highly non-linear equations with the help of special law of variation of Hubble's parameter, proposed by Bermann [2], that yields constant deceleration parameter models of the universe.

We consider only constant deceleration parameter model defined by

$$q = -\left[\frac{RR_{44}}{(R_4)^2}\right] = \text{constant} \tag{27}$$

where $R = (B^2 C e^{-ax})^{\frac{1}{3}}$ is the over all scale factor. Here the constant is taken as negative (i.e. it is an accelerating model of the universe). The solution of (27) is given by

$$R = (\alpha t + \beta)^{\frac{1}{1+q}} \tag{28}$$

where $\alpha \neq 0$ and β are constants of integration.

This equation implies that the condition of expansion is $1 + q > 0$.

Field equations (22–26) with the help of (28) and relation between metric potential $B = \mu C$, now admit an exact solution given by

$$A = B = \mu^{\frac{1}{3}} e^{\frac{ax}{3}} (\alpha t + \beta)^{\frac{1}{1+q}}, \tag{29}$$

$$C = \mu^{-\frac{2}{3}} e^{\frac{ax}{3}} (\alpha t + \beta)^{\frac{1}{1+q}}, \tag{30}$$

$$\phi = \left(\frac{1+q}{q-2}\right) K_1 \mu^2 e^{-ax} (\alpha t + \beta)^{\frac{q-2}{1+q}} + \phi_0, \tag{31}$$

$$\rho = 3p = K_2 e^{-\frac{4ax}{3}} (\alpha t + \beta)^{-\frac{4}{1+q}} \tag{32}$$

where K_1, K_2 and ϕ_0 are constants, $1 + q > 0$, $q \neq 0$.

Hence, Bianchi type-III radiating cosmological model corresponding to the above solution, can be written (through a proper choice of coordinates and constants of integration) as

$$ds^2 = dT^2 - \mu^{\frac{2}{3}} e^{\frac{2ax}{3}} T^{\frac{2}{1+q}} dX^2 - \mu^{\frac{2}{3}} e^{-\frac{4ax}{3}} T^{\frac{2}{1+q}} dY^2 - \mu^{-\frac{4}{3}} e^{\frac{2ax}{3}} T^{\frac{2}{1+q}} dZ^2. \tag{33}$$

3 Some Physical and Kinematical Properties

The model (33) represents an exact radiating cosmological model with a negative constant deceleration parameter (i.e. it is an accelerating universe) in the frame-work of Brans-Dicke scalar-tensor theory of gravitation.

The physical quantities that are important in cosmology are proper volume V^3, the expansion scalar θ, shear scalar σ^2 and Hubble's parameter H. They have the following expressions for the model (33):

$$\text{Special volume } V^3 = T^{\frac{3}{1+q}}, \tag{34}$$

$$\text{Scalar expansion } \theta = \frac{1}{3} u^i_{;i} = \frac{\alpha}{(1+q)T}, \tag{35}$$

$$\text{Shear scalar } \sigma^2 = \frac{1}{2} \sigma^{ij} \sigma_{ij} = \frac{\alpha^2}{6(1+q)^2} \frac{1}{T^2}, \tag{36}$$

$$\text{Hubble's parameter } H = \frac{R_4}{R} = \left(\frac{\alpha}{1+q} \right) \frac{1}{T}. \tag{37}$$

The expressions for the pressure p, energy density ρ of the fluid and scalar field ϕ in the model are given by

$$\begin{aligned} \rho &= 3p = K_2 e^{-\frac{4ax}{3}} (T)^{-\frac{4}{1+q}}, \\ \phi &= \left(\frac{1+q}{q-2} \right) e^{-ax} (T)^{\frac{q-2}{1+q}}. \end{aligned} \tag{38}$$

From (34–37) one can observe that at the initial epoch $T = 0$, all the physical quantities diverge, while the Brans-Dicke scalar field ϕ has no initial singularity. Thus the universe starts with an infinite rate of expansion and measure of anisotropy. So this is in accordance with big bang model. Also from (34–38) as $T \to \infty$, the proper volume becomes infinitely large and expansion scalar, shear scalar, density and pressure tend to zero.

Also, since $\lim_{T\to\infty}(\frac{\sigma^2}{\theta^2}) \neq 0$, the model does not approach isotropy for large value of T.

4 Conclusion

In this paper, we have considered Brans-Dicke [1] field equations in the presence of perfect fluid with disordered radiation for Bianchi type-III space time. For solving the field equations, we have used the special law of variation of Hubble's parameter proposed by Bermann [2]. The cosmological model, thus obtained, represents a radiating universe in Brans-Dicke theory of gravitation.

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